DESIGN AND ANALYSIS OF
EXPERIMENTS

TENTH EDITION

DOUGLAS C. MONTGOMERY

WILEY

Design and Analysis of Experiments

Tenth Edition

DOUGLAS C. MONTGOMERY
Arizona State University

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Preface

Audience

This is an introductory textbook dealing with the design and analysis of experiments. It is based on college-level courses in design of experiments that I have taught for over 40 years at Arizona State University, the University of Washington, and the Georgia Institute of Technology. It also reflects the methods that I have found useful in my own professional practice as an engineering and statistical consultant in many areas of science and engineering, including the research and development activities required for successful technology commercialization and product realization.

The book is intended for students who have completed a first course in statistical methods. This background course should include at least some techniques of descriptive statistics, the standard sampling distributions, and an introduction to basic concepts of confidence intervals and hypothesis testing for means and variances. Chapters 10, 11, and 12 require some familiarity with matrix algebra.

Because the prerequisites are relatively modest, this book can be used in a second course on statistics focusing on statistical design of experiments for undergraduate students in engineering, the physical and chemical sciences, statistics, mathematics, and other fields of science. For many years, I have taught a course from the book at the first-year graduate level in engineering. Students in this course come from all of the fields of engineering, materials science, physics, chemistry, mathematics, operations research life sciences, and statistics. I have also used this book as the basis of an industrial short course on design of experiments for practicing technical professionals with a wide variety of backgrounds. There are numerous examples illustrating all of the design and analysis techniques. These examples are based on real-world applications of experimental design and are drawn from many different fields of engineering and the sciences. This adds a strong applications flavor to an academic course for engineers and scientists and makes the book useful as a reference tool for experimenters in a variety of disciplines.

About the Book

The tenth edition is published for the first time as an enhanced e-text. The new edition adds media, some interactivity, and convenient direct access to supplemental material and data sets. In terms of content, the book has the same balance between design and analysis topics of previous editions. There continues to be a lot of emphasis on the computer in this edition.

Design-Expert, JMP, and Minitab Software

During the last few years, a number of excellent software products to assist experimenters in both the design and analysis phases of this subject have appeared. I have included output from three of these products, Design-Expert, JMP, and Minitab at many points in the text. Minitab and JMP are widely available general-purpose statistical software packages that have good data analysis capabilities and that handles the analysis of experiments with both fixed and random factors (including the mixed model). Design-Expert is a package focused exclusively on experimental design. All three of these packages have many capabilities for construction and evaluation of designs and extensive analysis features. I urge all instructors who use this book to incorporate computer software into your course. (In my course, I bring a laptop computer, and every design or analysis topic discussed in class is illustrated with the computer.)

Empirical Model

I have continued to focus on the connection between the experiment and the model that the experimenter can develop from the results of the experiment. Engineers (and physical, chemical, and life scientists to a large extent) learn about physical mechanisms and their underlying mechanistic models early in their academic training, and throughout much of their professional careers they are involved with manipulation of these models. Statistically designed experiments offer the engineer a valid basis for developing an *empirical* model of the system being investigated. This empirical model can then be manipulated (perhaps through a response surface or contour plot, or perhaps mathematically) just as any other engineering model. I have discovered through many years of teaching that this viewpoint is very effective in creating enthusiasm in the engineering community for statistically designed experiments. Therefore, the notion of an underlying empirical model for the experiment and response surfaces appears early in the book and continues to receive emphasis.

Factorial Designs

In the recently published ninth edition, I expanded the material on factorial and fractional factorial designs (Chapters 5–9) in an effort to make the material flow more effectively from both the reader's and the instructor's viewpoint and to place more emphasis on the empirical model. The new material includes follow-up experimentation following a fractional factorial, nonregular and nonorthogonal designs, and small, efficient resolution IV and V designs. Nonregular fractions as alternatives to traditional minimum aberration fractions in 16 runs and analysis methods for these design are discussed and illustrated.

Additional Important Changes

I also added new material on various other topics, including optimal designs and their application. The chapter on response surfaces (Chapter 11) has several new topics and problems. I expanded Chapter 12 on robust parameter design and process robustness experiments. Chapters 13 and 14 discuss experiments involving random effects and some applications of these concepts to nested and split-plot designs. The residual maximum likelihood method is now widely available in software, and I have emphasized this technique throughout the book. Because there is expanding industrial interest in nested and split-plot designs, Chapters 13 and 14 have several new topics. Chapter 15 is an overview of important design and analysis topics: nonnormality of the response, the Box–Cox method for selecting the form of a transformation, and other alternatives; unbalanced factorial experiments; the analysis of covariance, including covariates in a factorial design, and repeated measures. I also added new examples and problems from various fields, including biochemistry and biotechnology.

Experimental Design

Throughout the book, I have stressed the importance of experimental design as a tool for engineers and scientists to use for product design and development as well as process development and improvement. The use of experimental design in developing products that are robust to environmental factors and other sources of variability is illustrated. I believe that the use of experimental design early in the product cycle can substantially reduce development lead time and cost, leading to processes and products that perform better in the field and have higher reliability than those developed using other approaches.

The book contains more material than can be covered comfortably in one course, and I hope that instructors will be able to either vary the content of each course offering or discuss some topics in greater depth, depending on class interest. There are problem sets at the end of each chapter. These problems vary in scope from computational exercises, designed to reinforce the fundamentals, to extensions or elaboration of basic principles.

Course Suggestions

My own course focuses extensively on factorial and fractional factorial designs. Consequently, I usually cover Chapter 1, Chapter 2 (very quickly), most of Chapter 3, Chapter 4 (excluding the material on incomplete blocks and only mentioning Latin squares briefly), and I discuss Chapters 5 through 8 on factorials and two-level factorial and fractional factorial designs in detail. To conclude the course, I introduce response surface methodology (Chapter 11) and give an overview of random effects models (Chapter 13) and nested and split-plot designs (Chapter 14). I always require the students to complete a term project that involves designing, conducting, and presenting the results of a statistically designed experiment. I require them to do this in teams because this is the way that much industrial experimentation is conducted. They must present the results of this project, both orally and in written form.

The Supplemental Text Material

For this edition, I have provided supplemental text material for each chapter of the book. Often, this supplemental material elaborates on topics that could not be discussed in greater detail in the book. I have also presented some subjects that do not appear directly in the book, but an introduction to them could prove useful to some students and professional practitioners. Some of this material is at a higher mathematical level than the text. I realize that instructors use this book with a wide array of audiences, and some more advanced design courses could possibly benefit from including several of the supplemental text material topics. This material is available with the Study Resources in the e-text and also to instructors on the companion site for this book, located at www.wiley.com/go/montgomery/designandanalysisofexperiments10e.

Student and Instructor Supplements

Current supporting material for instructors is available at the website www.wiley.com/go/montgomery/designandanalysisofexperiments10e. This site will be used to communicate information about innovations and recommendations for effectively using this text. The supplemental text material described above is available at the site, along with electronic versions of data sets used for examples and homework problems, a course syllabus, and some representative student term projects from the course at Arizona State University. For students, the enhanced e-text has links to the supplemental text material and data sets.

Student Materials

The materials available to students via links in the e-text include the following:

1. The supplemental text material described above
2. Data sets from the book examples and homework problems, in electronic form

Sample Student Projects are available from instructors.

Instructor Materials

The instructor's section of the textbook website contains the following:

1. Solutions to the text problems
2. The supplemental text material described above
3. PowerPoint lecture slides
4. Figures from the text in electronic format, for easy inclusion in lecture slides
5. Data sets from the book examples and homework problems, in electronic form
6. Sample Syllabus
7. Sample Student Projects

The instructor's section is for instructor use only, and is password-protected. Visit the Instructor Companion Site portion of the website, located at www.wiley.com/go/montgomery/designandanalysisofexperiments10e, to register for a password.

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The contributions of the professional practitioners with whom I have worked have been invaluable. It is impossible to mention everyone, but some of the major contributors include Dr. Dan McCarville, Dr. Lisa Custer, Dr. Richard Post, Mr. Tom Bingham, Mr. Dick Vaughn, Dr. Julian Anderson, Mr. Richard Alkire, and Mr. Chase Neilson of the Boeing Company; Mr. Mike Goza, Mr. Don Walton, Ms. Karen Madison, Mr. Jeff Stevens, and Mr. Bob Kohm of Alcoa; Dr. Jay Gardiner, Mr. John Butora, Mr. Dana Lesher, Mr. Lolly Marwah, Mr. Leon Mason of IBM; Dr. Paul Tobias of IBM and Sematech; Ms. Elizabeth A. Peck of The Coca-Cola Company; Dr. Sadri Khalessi and Mr. Franz Wagner of Signetics; Mr. Robert V. Baxley of Monsanto Chemicals; Mr. Harry Peterson-Nedry and Dr. Russell Boyles of Precision Castparts Corporation; Mr. Bill New and Mr. Randy Schmid of Allied-Signal Aerospace; Mr. John M. Fluke, Jr. of the John Fluke Manufacturing Company; Mr. Larry Newton and Mr. Kip Howlett of Georgia-Pacific; and Dr. Ernesto Ramos of BBN Software Products Corporation.

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DOUGLAS C. MONTGOMERY
TEMPE, ARIZONA

Contents

OC Content available in eBook
SS Student solution available in interactive e-text

Chapter 1 Problems

SS Student solution available in interactive e-text.

SS **1.1** Suppose that you want to design an experiment to study the proportion of unpopped kernels of popcorn. Complete steps 1–3 of the guidelines for designing experiments in Section 1.4. Are there any major sources of variation that would be difficult to control?

1.2 Suppose that you want to investigate the factors that potentially affect cooking rice.

(a) What would you use as a response variable in this experiment? How would you measure the response?

(b) List all of the potential sources of variability that could impact the response.

(c) Complete the first three steps of the guidelines for designing experiments in Section 1.4.

1.3 Suppose that you want to compare the growth of garden flowers with different conditions of sunlight, water, fertilizer, and soil conditions. Complete steps 1–3 of the guidelines for designing experiments in Section 1.4.

1.4 Select an experiment of interest to you. Complete steps 1–3 of the guidelines for designing experiments in Section 1.4.

1.5 Search the World Wide Web for information about Sir Ronald A. Fisher and his work on experimental design in agricultural science at the Rothamsted Experimental Station.

1.6 Find a website for a business that you are interested in. Develop a list of factors that you would use in an experiment to improve the effectiveness of this website.

1.7 Almost everyone is concerned about the price of gasoline. Construct a cause-and-effect diagram identifying the factors that potentially influence the gasoline mileage that you get in your car. How would you go about conducting an experiment to determine any of these factors actually affect your gasoline mileage?

1.8 What is replication? Why do we need replication in an experiment? Present an example that illustrates the difference between replication and repeated measurements.

1.9 Why is randomization important in an experiment? SS

1.10 What are the potential risks of a single, large, comprehensive experiment in contrast to a sequential approach? SS

CHAPTER 1

Introduction

CHAPTER LEARNING OBJECTIVES

1. Learn about the objectives of experimental design and the role it plays in the knowledge discovery process.
2. Learn about different strategies of experimentation.
3. Understand the role that statistical methods play in designing and analyzing experiments.
4. Understand the concepts of main effects of factors and interaction between factors.
5. Know about factorial experiments.
6. Know the practical guidelines for designing and conducting experiments.

1.1 Strategy of Experimentation

Observing a system or process while it is in operation is an important part of the learning process and is an integral part of understanding and learning about how systems and processes work. The great New York Yankees catcher Yogi Berra said that "... you can observe a lot just by watching." However, to understand what happens to a process when you change certain input factors, you have to do more than just watch—you actually have to change the factors. This means that to really understand cause-and-effect relationships in a system you must deliberately change the input variables to the system and observe the changes in the system output that these changes to the inputs produce. In other words, you need to conduct **experiments** on the system. Observations on a system or process can lead to theories or hypotheses about what makes the system work, but experiments of the type described above are required to demonstrate that these theories are correct.

Investigators perform experiments in virtually all fields of inquiry, usually to discover something about a particular process or system or to confirm previous experience or theory. Each experimental **run** is a **test**. More formally, we can define an **experiment** as a test or series of runs in which purposeful changes are made to the input variables of a process or system so that we may observe and identify the reasons for changes that may be observed in the output response. We may want to determine which input variables are responsible for the observed changes in the response, develop a model relating the response to the important input variables, and use this model for process or system improvement or other decision-making.

This book is about planning and conducting experiments and about analyzing the resulting data so that valid and objective conclusions are obtained. Our focus is on experiments in engineering and science. Experimentation plays an important role in **technology commercialization** and **product realization** activities, which consist of new product design and formulation, manufacturing process development, and process improvement. The objective in many cases may be to develop a **robust** process, that is, a process affected minimally by external sources of variability. There are also many applications of designed experiments in a nonmanufacturing or non-product-development setting, such as marketing, service operations, and general business operations. Designed experiments are a key technology for **innovation**. Both **breakthrough innovation** and **incremental innovation** activities can benefit from the effective use of designed experiments.

As an example of an experiment, suppose that a metallurgical engineer is interested in studying the effect of two different hardening processes, oil quenching and saltwater quenching, on an aluminum alloy. Here the objective of the **experimenter** (the engineer) is to determine which quenching solution produces the maximum hardness for this particular alloy. The engineer decides to subject a number of alloy specimens or test coupons to each quenching medium and measure the hardness of the specimens after quenching. The average hardness of the specimens treated in each quenching solution will be used to determine which solution is best.

As we consider this simple experiment, a number of important questions come to mind:

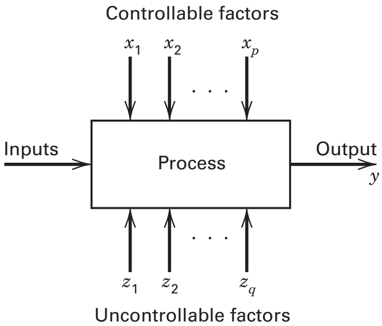
1. Are these two solutions the only quenching media of potential interest?
2. Are there any other factors that might affect hardness that should be investigated or controlled in this experiment (such as the temperature of the quenching media)?
3. How many coupons of alloy should be tested in each quenching solution?
4. How should the test coupons be assigned to the quenching solutions, and in what order should the data be collected?
5. What method of data analysis should be used?
6. What difference in average observed hardness between the two quenching media will be considered important?

All of these questions, and perhaps many others, will have to be answered satisfactorily before the experiment is performed.

Experimentation is a vital part of the **scientific** (or **engineering**) **method**. Now there are certainly situations where the scientific phenomena are so well understood that useful results including mathematical models can be developed directly by applying these well-understood principles. The models of such phenomena that follow directly from the physical mechanism are usually called **mechanistic models**. A simple example is the familiar equation for current flow in an electrical circuit, Ohm's law, $E = IR$. However, most problems in science and engineering require **observation** of the system at work and **experimentation** to elucidate information about why and how it works. Well-designed experiments can often lead to a model of system performance; such experimentally determined models are called **empirical models**. Throughout this book, we will present techniques for turning the results of a designed experiment into an empirical model of the system under study. These empirical models can be manipulated by a scientist or an engineer just as a mechanistic model can.

A well-designed experiment is important because the results and conclusions that can be drawn from the experiment depend to a large extent on the manner in which the data were collected. To illustrate this point, suppose that the metallurgical engineer in the above experiment used specimens from one heat in the oil quench and specimens from a second heat in the saltwater quench. Now, when the mean hardness is compared, the engineer is unable to say how much of the observed difference is the result of the quenching media and how much is the result of inherent differences between the heats.[1] Thus, the method of data collection has adversely affected the conclusions that can be drawn from the experiment.

[1] A specialist in experimental design would say that the effects of quenching media and heat were *confounded*; that is, the effects of these two factors cannot be separated.



■ **FIGURE 1.1** **General model of a process or system**

In general, experiments are used to study the performance of processes and systems. The process or system can be represented by the model shown in Figure 1.1. We can usually visualize the process as a combination of operations, machines, methods, people, and other resources that transforms some input (often a material) into an output that has one or more observable **response** variables. Some of the process variables and material properties $x_1, x_2, \ldots, x_p$ are **controllable**, whereas other variables such as environmental factors or some material properties $z_1, z_2, \ldots, z_q$ are **uncontrollable** (although they may be controllable for purposes of a test). The objectives of the experiment may include the following:

1. Determining which variables are most influential on the response y
2. Determining where to set the influential x's so that y is almost always near the desired nominal value
3. Determining where to set the influential x's so that variability in y is small
4. Determining where to set the influential x's so that the effects of the uncontrollable variables $z_1, z_2, \ldots, z_q$ are minimized.

As you can see from the foregoing discussion, experiments often involve several factors. Usually, an objective of the **experimenter** is to determine the influence that these factors have on the output response of the system. The general approach to planning and conducting the experiment is called the **strategy of experimentation**. An experimenter can use several strategies. We will illustrate some of these with a very simple example.

I really like to play golf. Unfortunately, I do not enjoy practicing, so I am always looking for a simpler solution to lowering my score. Some of the factors that I think may be important, or that may influence my golf score, are as follows:

1. The type of driver used (oversized or regular sized)
2. The type of ball used (balata or three piece)
3. Walking and carrying the golf clubs or riding in a golf cart
4. Drinking water or drinking "something else" while playing
5. Playing in the morning or playing in the afternoon
6. Playing when it is cool or playing when it is hot
7. The type of golf shoe spike worn (metal or soft)
8. Playing on a windy day or playing on a calm day.

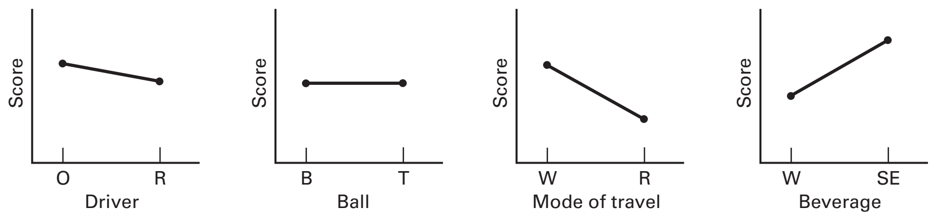
Obviously, many other factors could be considered, but let's assume that these are the ones of primary interest. Furthermore, based on long experience with the game, I decide that factors 5 through 8 can be ignored; that is, these factors are not important because their effects are so small that they have no practical value. Engineers, scientists, and business analysts often must make these types of decisions about some of the factors they are considering in real experiments.

Now, let's consider how factors 1 through 4 could be experimentally tested to determine their effect on my golf score. Suppose that a maximum of eight rounds of golf can be played over the course of the experiment. One approach would be to select an arbitrary combination of these factors, test them, and see what happens. For example, suppose the oversized driver, balata ball, golf cart, and water combination is selected, and the resulting score is 87. During the round, however, I noticed several wayward shots with the big driver (long is not always good in golf), and, as a result, I decide to play another round with the regular-sized driver, holding the other factors at the same levels used previously. This approach could be continued almost indefinitely, switching the levels of one or two (or perhaps several) factors for the next test, based on the outcome of the current test. This strategy of experimentation, which we call the **best-guess approach**, is frequently used in practice by engineers and scientists. It often works reasonably well, too, because the experimenters often have a great deal of technical or theoretical knowledge of the system they are studying, as well as considerable practical experience. The best-guess approach has at least two disadvantages. First, suppose the initial best-guess does not produce the desired results. Now the experimenter has to take another guess at the correct combination of factor levels. This could continue for a long time, without any guarantee of success. Second, suppose the initial best-guess produces an acceptable result. Now the experimenter is tempted to stop testing, although there is no guarantee that the *best* solution has been found.

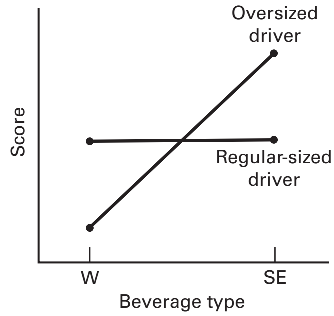
Another strategy of experimentation that is used extensively in practice is the **one-factor-at-a-time** (**OFAT**) approach. The OFAT method consists of selecting a starting point, or **baseline** set of levels, for each factor, and then successively varying each factor over its range with the other factors held constant at the baseline level. After all tests are performed, a series of graphs are usually constructed showing how the response variable is affected by varying each factor with all other factors held constant. Figure 1.2 shows a set of these graphs for the golf experiment, using the oversized driver, balata ball, walking, and drinking water levels of the four factors as the baseline. The interpretation of these graphs is straightforward; for example, because the slope of the mode of travel curve is negative, we would conclude that riding improves the score. Using these OFAT graphs, we would select the optimal combination to be the regular-sized driver, riding, and drinking water. The type of golf ball seems unimportant.

The major disadvantage of the OFAT strategy is that it fails to consider any possible **interaction** between the factors. An interaction is the failure of one factor to produce the same effect on the response at different levels of another factor. Figure 1.3 shows an interaction between the type of driver and the beverage factors for the golf experiment. Notice that if I use the regular-sized driver, the type of beverage consumed has virtually no effect on the score, but if I use the oversized driver, much better results are obtained by drinking water instead of "something else." Interactions between factors are very common, and if they occur, the OFAT strategy will usually produce poor results. Many people do not recognize this, and, consequently, OFAT experiments are run frequently in practice. (Some individuals actually think that this strategy is related to the scientific method or that it is a "sound" engineering principle.) OFAT experiments are always less efficient than other methods based on a statistical approach to design. We will discuss this in more detail in Chapter 5.

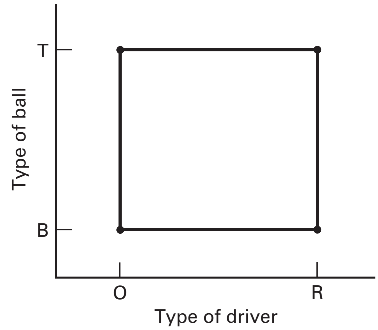
The correct approach to dealing with several factors is to conduct a **factorial** experiment. This is an experimental strategy in which factors are varied *together*, instead of one at a time. The factorial experimental design concept is extremely important, and several chapters in this book are devoted to presenting basic factorial experiments and a number of useful variations and special cases.



■ **FIGURE 1.2 Results of the one-factor-at-a-time strategy for the golf experiment**



■ **FIGURE 1.3**
Interaction between type of driver and type of beverage for the golf experiment



■ **FIGURE 1.4 A two-factor factorial experiment involving type of driver and type of ball**

To illustrate how a factorial experiment is conducted, consider the golf experiment and suppose that only two factors, type of driver and type of ball, are of interest. Figure 1.4 shows a two-factor factorial experiment for studying the joint effects of these two factors on my golf score. Notice that this factorial experiment has both factors at two levels and that all possible combinations of the two factors across their levels are used in the design. Geometrically, the four runs form the corners of a square. This particular type of factorial experiment is called a **2^2 factorial design** (two factors, each at two levels). Because I can reasonably expect to play eight rounds of golf to investigate these factors, a reasonable plan would be to play two rounds of golf at each combination of factor levels shown in Figure 1.4. An experimental designer would say that we have **replicated** the design twice. This experimental design would enable the experimenter to investigate the individual effects of each factor (or the **main** effects) and to determine whether the factors interact.

Figure 1.5*a* shows the results of performing the factorial experiment in Figure 1.4. The scores from each round of golf played at the four test combinations are shown at the corners of the square. Notice that there are four rounds of golf that provide information about using the regular-sized driver and four rounds that provide information about using the oversized driver. By finding the average difference in the scores on the right- and left-hand sides of the square (as in Figure 1.5*b*), we have a measure of the effect of switching from the oversized driver to the regular-sized driver, or

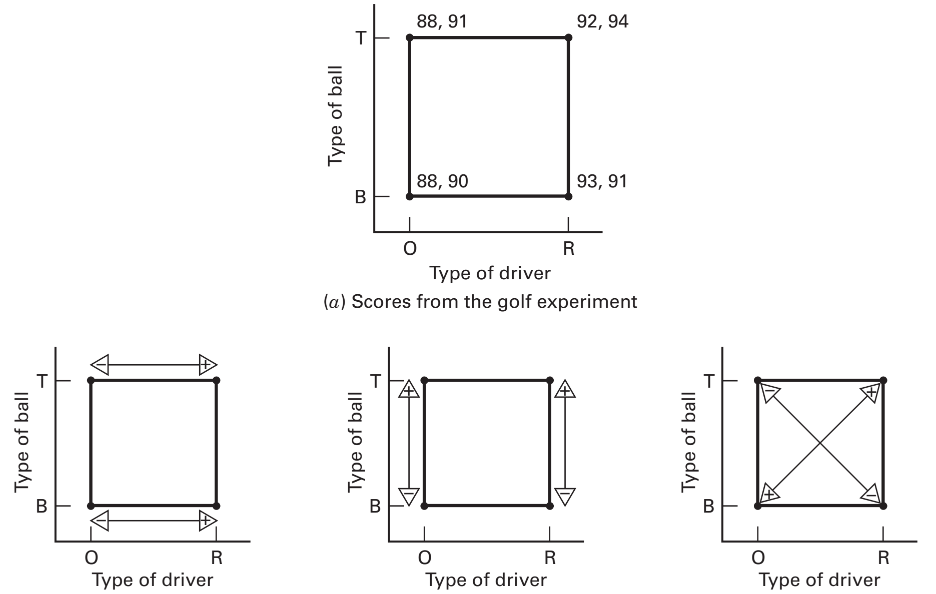
$$\begin{aligned}\text{Driver effect} &= \frac{92+94+93+91}{4} - \frac{88+91+88+90}{4}\\ &= 3.25\end{aligned}$$

That is, on average, switching from the oversized to the regular-sized driver increases the score by 3.25 strokes per round. Similarly, the average difference in the four scores at the top of the square and the four scores at the bottom measures the effect of the type of ball used (see Figure 1.5*c*):

$$\begin{aligned}\text{Ball effect} &= \frac{88+91+92+94}{4} - \frac{88+90+93+91}{4}\\ &= 0.75\end{aligned}$$

Finally, a measure of the interaction effect between the type of ball and the type of driver can be obtained by subtracting the average scores on the left-to-right diagonal in the square from the average scores on the right-to-left diagonal (see Figure 1.5*d*), resulting in

$$\begin{aligned}\text{Ball–driver interaction effect} &= \frac{92+94+88+90}{4} - \frac{88+91+93+91}{4}\\ &= 0.25\end{aligned}$$



(*a*) Scores from the golf experiment

(*b*) Comparison of scores leading to the driver effect

(*c*) Comparison of scores leading to the ball effect

(*d*) Comparison of scores leading to the ball–driver interaction effect

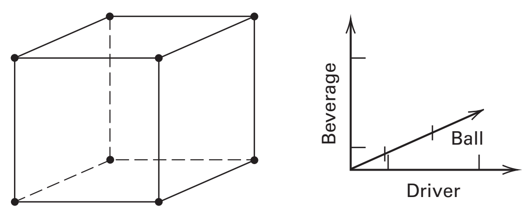
■ **FIGURE 1.5 Scores from the golf experiment in Figure 1.4 and calculation of the factor effects**

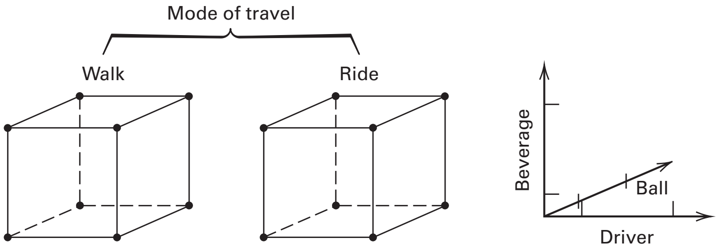
The results of this factorial experiment indicate that driver effect is larger than either the ball effect or the interaction. Statistical testing could be used to determine whether any of these effects differ from zero. In fact, it turns out that there is reasonably strong statistical evidence that the driver effect differs from zero and the other two effects do not. Therefore, this experiment indicates that I should always play with the oversized driver.

One very important feature of the factorial experiment is evident from this simple example; namely, factorials make the most efficient use of the experimental data. Notice that this experiment included eight observations, and all eight observations are used to calculate the driver, ball, and interaction effects. No other strategy of experimentation makes such an efficient use of the data. This is an important and useful feature of factorials.

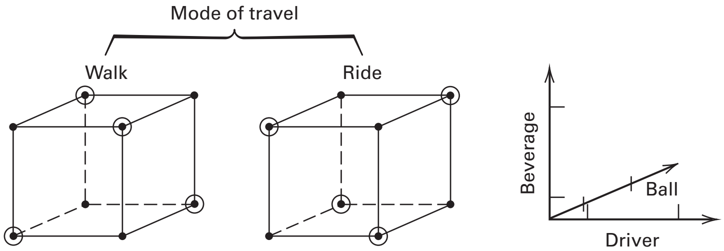
We can extend the factorial experiment concept to three factors. Suppose that I wish to study the effects of type of driver, type of ball, and the type of beverage consumed on my golf score. Assuming that all three factors have two levels, a factorial design can be set up as shown in Figure 1.6. Notice that there are eight test combinations of these three factors across the two levels of each and that these eight trials can be represented geometrically as the corners of a cube. This is an example of a **2^3 factorial design**. Because I only want to play eight rounds of golf, this experiment would require that one round be played at each combination of factors represented by the eight corners of the cube in Figure 1.6. However, if we compare this to the two-factor factorial in Figure 1.4, the 2^3 factorial design would provide the same information about the factor effects. For example, there are four tests in both designs that provide information about the regular-sized driver and four tests that provide information about the oversized driver, assuming that each run in the two-factor design in Figure 1.4 is replicated twice.

■ **FIGURE 1.6 A three-factor factorial experiment involving type of driver, type of ball, and type of beverage**





■ **FIGURE 1.7** **A four-factor factorial experiment involving type of driver, type of ball, type of beverage, and mode of travel**



■ **FIGURE 1.8** **A four-factor fractional factorial experiment involving type of driver, type of ball, type of beverage, and mode of travel**

Figure 1.7 illustrates how all four factors—driver, ball, beverage, and mode of travel (walking or riding)—could be investigated in a **2^4 factorial design**. As in any factorial design, all possible combinations of the levels of the factors are used. Because all four factors are at two levels, this experimental design can still be represented geometrically as a cube (actually a hypercube).

Generally, if there are k factors, each at two levels, the factorial design would require 2^k runs. For example, the experiment in Figure 1.7 requires 16 runs. Clearly, as the number of factors of interest increases, the number of runs required increases rapidly; for instance, a 10-factor experiment with all factors at two levels would require 1024 runs. This quickly becomes infeasible from a time and resource viewpoint. In the golf experiment, I can only play eight rounds of golf, so even the experiment in Figure 1.7 is too large.

Fortunately, if there are four to five or more factors, it is usually unnecessary to run all possible combinations of factor levels. A **fractional factorial experiment** is a variation of the basic factorial design in which only a subset of the runs is used. Figure 1.8 shows a fractional factorial design for the four-factor version of the golf experiment. This design requires only 8 runs instead of the original 16 and would be called a **one-half fraction**. If I can play only eight rounds of golf, this is an excellent design in which to study all four factors. It will provide good information about the main effects of the four factors as well as some information about how these factors interact.

Fractional factorial designs are used extensively in industrial research and development, and for process improvement. These designs will be discussed in Chapters 8 and 9.

1.2 Some Typical Applications of Experimental Design

Experimental design methods have found broad application in many disciplines. As noted previously, we may view experimentation as part of the scientific process and as one of the ways by which we learn about how systems or processes work. Generally, we learn through a series of activities in which we make conjectures about a process, perform experiments to generate data from the process, and then use the information from the experiment to establish new conjectures, which lead to new experiments, and so on.

Experimental design is a critically important tool in the scientific and engineering world for driving innovation in the product realization process. Critical components of these activities are in new manufacturing process design and development and process management. The application of experimental design techniques early in process development can result in

1. Improved process yields
2. Reduced variability and closer conformance to nominal or target requirements
3. Reduced development time
4. Reduced overall costs.

Experimental design methods are also of fundamental importance in **engineering design** activities, where new products are developed and existing ones improved. Some applications of experimental design in engineering design include

1. Evaluation and comparison of basic design configurations
2. Evaluation of material alternatives
3. Selection of design parameters so that the product will work well under a wide variety of field conditions, that is, so that the product is **robust**
4. Determination of key product design parameters that impact product performance
5. Formulation of new products.

The use of experimental design in product realization can result in products that are easier to manufacture and that have enhanced field performance and reliability, lower product cost, and shorter product design and development time. Designed experiments also have extensive applications in marketing, market research, transactional and service operations, and general business operations. We now present several examples that illustrate some of these ideas.

EXAMPLE 1.1 Characterizing a Process

A flow solder machine is used in the manufacturing process for printed circuit boards. The machine cleans the boards in a flux, preheats the boards, and then moves them along a conveyor through a wave of molten solder. This solder process makes the electrical and mechanical connections for the leaded components on the board.

The process currently operates around the 1 percent defective level. That is, about 1 percent of the solder joints on a board are defective and require manual retouching. However, because the average printed circuit board contains over 2000 solder joints, even a 1 percent defective level results in far too many solder joints requiring rework. The process engineer responsible for this area would like to use a designed experiment to determine which machine parameters are influential in the occurrence of solder defects and which adjustments should be made to those variables to reduce solder defects.

The flow solder machine has several variables that can be controlled. They include

1. Solder temperature
2. Preheat temperature
3. Conveyor speed
4. Flux type
5. Flux specific gravity
6. Solder wave depth
7. Conveyor angle.

In addition to these controllable factors, several other factors cannot be easily controlled during routine manufacturing, although they could be controlled for the purposes of a test. They are

1. Thickness of the printed circuit board
2. Types of components used on the board
3. Layout of the components on the board
4. Operator
5. Production rate.

In this situation, engineers are interested in **characterizing** the flow solder machine; that is, they want to determine which factors (both controllable and uncontrollable) affect the occurrence of defects on the printed circuit boards. To accomplish this, they can design an experiment that will enable them to estimate the magnitude and direction of the factor effects; that is, how much does the response variable (defects per unit) change when each factor is changed, and does changing the factors *together* produce different results than are obtained from individual factor adjustments—that is, do the factors interact? Sometimes we call an experiment such as this a **screening experiment**. Typically, screening or characterization experiments involve using fractional factorial designs, such as in the golf example in Figure 1.8.

The information from this screening or characterization experiment will be used to identify the critical process factors and to determine the direction of adjustment for these factors to reduce further the number of defects per unit. The experiment may also provide information about which factors should be more carefully controlled during routine manufacturing to prevent high defect levels and erratic process performance. Thus, one result of the experiment could be the application of techniques such as control charts to one or more **process variables** (such as solder temperature), in addition to control charts on process output. Over time, if the process is improved enough, it may be possible to base most of the process control plan on controlling process input variables instead of control charting the output.

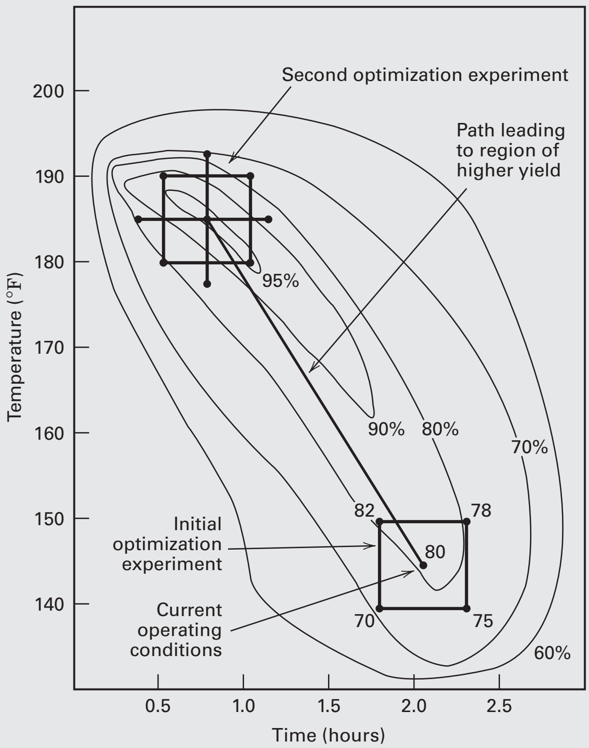
EXAMPLE 1.2 Optimizing a Process

In a characterization experiment, we are usually interested in determining which process variables affect the response. A logical next step is to optimize, that is, to determine the region in the important factors that leads to the best possible response. For example, if the response is yield, we would look for a region of maximum yield, whereas if the response is variability in a critical product dimension, we would seek a region of minimum variability.

Suppose that we are interested in improving the yield of a chemical process. We know from the results of a characterization experiment that the two most important process variables that influence the yield are operating temperature and reaction time. The process currently runs at 145°F and 2.1 hours of reaction time, producing yields of around 80 percent. Figure 1.9 shows a view of the time–temperature region from above. In this graph, the lines of constant yield are connected to form response **contours**, and we have shown the contour lines for yields of 60, 70, 80, 90, and 95 percent. These contours are projections on the time–temperature region of cross sections of the yield surface corresponding to the aforementioned percent yields. This surface is sometimes called a **response surface**. The true response surface in Figure 1.9 is unknown to the process personnel, so experimental methods will be required to optimize the yield with respect to time and temperature.

To locate the optimum, it is necessary to perform an experiment that varies both time and temperature together, that is, a factorial experiment. The results of an initial factorial experiment with both time and temperature run at two levels are shown in Figure 1.9. The responses observed at the four corners of the square indicate that we should move in the general direction of increased temperature and decreased reaction time to increase yield. A few additional runs would be performed in this direction, and this additional experimentation would lead us to the region of maximum yield.

Once we have found the region of the optimum, a second experiment would typically be performed. The objective of this second experiment is to develop an empirical model of the process and to obtain a more precise estimate of the optimum operating conditions for time and temperature. This approach to process optimization is called **response surface methodology**, and it is explored in detail in Chapter 11. The second design illustrated in Figure 1.9 is a **central composite design**, one of the most important experimental designs used in process optimization studies.



■ **FIGURE 1.9 Contour plot of yield as a function of reaction time and reaction temperature, illustrating experimentation to optimize a process**